Identification Method for Lightly Damped Structures

Nelson G. Creamer*
General Research Corporation, Arlington, Virginia
and
John L. Junkins†
Texas A&M University, College Station, Texas

A structural model identification method is developed for determination of the mass and stiffness matrices of an undamped structure along with the damping matrix of a lightly damped structure. Using measurements of natural frequencies, damping factors, and frequency response elements, a unique identification of the model is established through incorporation of the spectral decomposition of the frequency response function and the modal orthonormality conditions. Numerical simulations demonstrate the flexibility and potential of the proposed method.

Introduction

ACCURATE knowledge of the mass, damping, and stiffness associated with a dynamical system is a key ingredient for correlating theoretical and experimental results and for designing active control schemes for vibration suppression and attitude maneuvering. Discretization of a linear continuous structure by means of finite-element analysis (or other similar methods) yields the well-known mass and stiffness matrices. Although this discretization process may be well defined, the resulting structural model will be only as accurate as the parameters and modeling assumptions used to characterize the structural behavior. Also, determination of the damping matrix requires knowledge of parameters which may be difficult, if not impossible, to measure in the laboratory.

Methods for refining a priori structural models are readily available in the literature. References 1–6 address the identification of a set of physical/geometrical parameters using nonlinear least-squares and Bayesian estimation methods. The disadvantages of these methods are the following:

1) Use of natural frequencies and/or mode shapes, exclusively, results in nonunique identification of the initial parameterized model (in the sense that an infinity of linear models can produce the same set of eigenvalues and eigenvectors), unless some parameters are "fixed" at their initial values.

2) Convergence of the nonlinear estimation algorithms requires initial parameter estimates to be "close" to their true values.

References 7–11 determine mass and stiffness matrix improvements to enforce exact agreement between theory and experiment. Again, use of modal information alone results in both nonunique solutions and physically unrealistic coupling. Reference 12 utilizes submatrix scale factors to improve the initial mass and stiffness matrices using modal information, with the uniqueness problem once again surfacing. In Refs. 13–15, a linear algorithm is used to identify the mass, damping, and stiffness matrices from forced time-domain response. Although there is no initial estimate required for the model and the uniqueness problem is, in principle, eliminated, the disadvantages are now that the order of the resulting model is dependent on the number of sensors used on the structure and that the parameter vector consists of every element of the highly redundant mass, damping, and stiffness matrices.

A method for identifying the mass, damping, and stiffness matrices of an undamped or lightly damped structure using measured modal information and frequency response elements is developed in this paper. This method is designed to eliminate the problems described above and is simple to implement.

Identification of Undamped Structures

Consider the classical second-order equations governing the motion of an undamped structural system

$$M\ddot{u} + Ku = f \tag{1}$$

where M and K are the $n \times n$ mass and stiffness matrices, $\boldsymbol{u}$ is the $n \times 1$ generalized coordinate vector, and f is the $n \times 1$ generalized force vector. The initial estimates of the mass and stiffness matrices, $\hat{M}$ and $\hat{K}$, are obtained from a standard discretization process, i.e., the finite-element method. It is assumed that the following measurements, extracted from response of the actual structure, are available: 1) a set of m ($<n$) natural frequencies $\tilde{\omega}_i$, 2) a set of corresponding $n \times 1$ mode shapes $\tilde{\psi}_i$ (or approximations from the initial structural model), and 3) a small set of frequency response elements $\tilde{h}_{jk}(\omega)$ measured throughout the frequency range of interest for the structure. (A complete row/column of the frequency response matrix is not required.) Ewins[16] provides a review of many frequency- and time-domain approaches for generating these measurements. Ewins and Gleeson[17] developed a technique for obtaining modal measurements for lightly damped structures. Juang[18] provides a review of frequency- and time-domain modal identification techniques using system realization theory. The goal of the structural model identification method is to improve the initial mass and stiffness matrices such that the theoretical and experimental results are in agreement.

To begin, it is desirable to introduce the well-known spectral decomposition of the frequency response function

$$h_{jk}(\omega) = \sum_{r=1}^{n} \left(\frac{\phi_{jr}\phi_{kr}}{\omega_r^2 - \omega^2} \right) \tag{2}$$

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*Technical Staff. Member AIAA.

†TEES Chair Professor, Aerospace Engineering. Fellow AIAA.

where ϕ_{jr} is the jth element of the rth mass-normalized mode shape that, in matrix form, satisfies

$$\phi^T M \phi = I \tag{3}$$

Since the true mass matrix is not known, an approximation to Eq. (2) must be utilized. Introducing the relation

$$\phi_{jr} = \sqrt{\alpha_r}\tilde{\psi}_{jr}, \qquad r = 1,2,\ldots,m \tag{4}$$

into Eq. (2), and motivated by Ewins,[16] an approximation of the spectral decomposition can be written as

$$\tilde{h}_{jk}(\omega) = \frac{a_1}{\omega^2} + \sum_{r=1}^{m}\left(\frac{\tilde{\psi}_{jr}\tilde{\psi}_{kr}}{\tilde{\omega}_r^2 - \omega^2}\alpha_r\right) + a_2 \tag{5}$$

In Eq. (5), the first term represents the contribution from any rigid-body modes ($\tilde{\omega}_r = 0$), the last term represents an approximate residual contribution from high-frequency modes (outside the measured frequency range), and the α are to-be-determined modal normalization factors. By "sampling" throughout the frequency range of interest (N represents the number of samples), Eq. (5) can be rearranged into the following standard linear format to identify the modal normalization factors by the method of least squares

$$\begin{Bmatrix} \tilde{h}_{jk}(\omega_1) \\ \vdots \\ \tilde{h}_{jk}(\omega_N) \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{\omega_1^2} & L_{11}L_{12}\ldots L_{1m} & 1 \\ \vdots & \ddots & \vdots \\ \dfrac{1}{\omega_N^2} & L_{N1}L_{N2}\ldots L_{Nm} & 1 \end{bmatrix} \begin{Bmatrix} a_1 \\ \alpha_1 \\ \vdots \\ \alpha_m \\ a_2 \end{Bmatrix} \tag{6a}$$

where

$$L_{pq} = \frac{\tilde{\psi}_{jq}\tilde{\psi}_{kq}}{\tilde{\omega}_q^2 - \omega_p^2} \tag{6b}$$

To include measurements from more than one frequency response element, if available, a simple augmentation (or "stacking") of Eqs. (6) is required.

Once the modal normalization factors have been determined, the orthonormality conditions that the mode shapes must satisfy can be written as

$$\tilde{\psi}_i^T M \tilde{\psi}_j = \delta_{ij}/\alpha_j \tag{7a}$$

$$\tilde{\psi}_i^T K \tilde{\psi}_j = \delta_{ij}\tilde{\omega}_j^2/\alpha_j \tag{7b}$$

To identify the true mass and stiffness matrices, the following expansions are used[12]

$$M = \hat{M} + \sum_{r=1}^{P} \gamma_r M_r \tag{8a}$$

$$K = \hat{K} + \sum_{r=1}^{Q} \beta_r K_r \tag{8b}$$

where M_r and K_r are the rth predetermined mass and stiffness submatrices, γ_r and β_r the to-be-determined rth mass and stiffness submatrix scale factors, and P and Q the total number of mass and stiffness submatrices. The mass and stiffness submatrices can represent single finite elements or (more commonly) groups of common finite elements assembled into their corresponding global locations. The flexibility (and responsibility) in defining M_r and K_r in Eqs. (8) is an important feature that can be used to exploit an engineer's insight explicitly. Substituting Eqs. (8) into Eqs. (7) and re-arranging terms yields

$$\begin{Bmatrix} -\tilde{\psi}_i^T \hat{M} \tilde{\psi}_i + \dfrac{1}{\alpha_i} \\ -\tilde{\psi}_i^T \hat{M} \tilde{\psi}_j \end{Bmatrix} = \begin{bmatrix} \tilde{\psi}_i^T M_1 \tilde{\psi}_i & \ldots & \tilde{\psi}_i^T M_P \tilde{\psi}_i \\ \tilde{\psi}_i^T M_1 \tilde{\psi}_j & \ldots & \tilde{\psi}_i^T M_P \tilde{\psi}_j \end{bmatrix} \begin{Bmatrix} \gamma_1 \\ \vdots \\ \gamma_P \end{Bmatrix} \tag{9a}$$

$$\begin{Bmatrix} -\tilde{\psi}_i^T \hat{K} \tilde{\psi}_i + \dfrac{\tilde{\omega}_i^2}{\alpha_i} \\ -\tilde{\psi}_i^T \hat{K} \tilde{\psi}_j \end{Bmatrix} = \begin{bmatrix} \tilde{\psi}_i^T K_1 \tilde{\psi}_i & \ldots & \tilde{\psi}_i^T K_Q \tilde{\psi}_i \\ \tilde{\psi}_i^T K_1 \tilde{\psi}_j & \ldots & \tilde{\psi}_i^T K_Q \tilde{\psi}_j \end{bmatrix} \begin{Bmatrix} \beta_1 \\ \vdots \\ \beta_Q \end{Bmatrix} \tag{9b}$$

where the second set of equations in Eqs. (9a) and (9b) are valid when $i \neq j$. Collecting Eqs. (9) for each measured natural frequency yields a set of equations (linear in the unknown submatrix scale factors), which can be solved by a least-squares method, provided that $m(m+1)/2$ is greater than $\max(P,Q)$. Since Eq. (5) represents an approximation to the frequency response function, an iterative procedure can be used whereby the unmeasured natural frequencies and mode shapes are predicted from the present best estimate of the structural model and used in Eq. (5) in lieu of measurements.

Identification of Lightly Damped Structures

If a small amount of damping is present in a structure, the structural identification method developed in the previous section can be used, in conjunction with matrix perturbation theory, to identify the mass, damping, and stiffness matrices. Consider the symmetrical state-space representation of Eq. (1) in the form

$$A\begin{Bmatrix} \dot{u} \\ \ddot{u} \end{Bmatrix} + B\begin{Bmatrix} u \\ \dot{u} \end{Bmatrix} = \begin{Bmatrix} 0 \\ f \end{Bmatrix} \tag{10a}$$

where

$$A = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} \tag{10b}$$

If light viscous damping is introduced into the equations of motion in the form of the symmetric damping matrix C, the state-space representation is perturbed by the relation

$$B = \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix} = B_0 + B_1 \tag{11}$$

where B_1 represents the perturbation matrix due to the presence of the light damping. A first-order perturbation solution to the free-response eigenvalue problem of Eqs. (10) can be obtained to approximate the change in the eigenvalues due to the inclusion of the damping matrix.[19]

Consider the eigenvalue problem

$$-\lambda_{0r} A_0 \Phi_{0r} = B_0 \Phi_{0r} \tag{12}$$

where A_0 and B_0 are $2n \times 2n$ symmetric matrices and λ_{0r} and Φ_{0r} ($r = 1,2,\ldots,2n$) are the eigenvalues and eigenvectors. It is assumed that the eigenvectors are normalized such that

$$\Phi_{0j}^T A_0 \Phi_{0k} = \delta_{jk} \tag{13a}$$

$$\Phi_{0j}^T B_0 \Phi_{0k} = -\lambda_{0k}\delta_{jk} \tag{13b}$$

If small perturbations A_1 and B_1 are added to each matrix, the resulting eigenvalue problem becomes

$$-\lambda_r A \Phi_r = B \Phi_r \tag{14a}$$

where

$$A = A_0 + A_1 \tag{14b}$$

$$B = B_0 + B_1 \tag{14c}$$

$$\lambda_r = \lambda_{0r} + \lambda_{1r} \tag{14d}$$

$$\mathbf{\Phi}_r = \mathbf{\Phi}_{0r} + \mathbf{\Phi}_{1r} \tag{14e}$$

The eigenvalues λ_{1r} and eigenvectors $\mathbf{\Phi}_{1r}$ represent small perturbations from their original values. Expanding Eq. (14a), using Eq. (12), and neglecting second-order terms yields the equation

$$B_1\mathbf{\Phi}_{0r} + B_0\mathbf{\Phi}_{1r} = -\lambda_{0r}A_1\mathbf{\Phi}_{0r} - \lambda_{0r}A_0\mathbf{\Phi}_{1r} - \lambda_{1r}A_0\mathbf{\Phi}_{0r} \tag{15}$$

Multiplying Eq. (15) by $\mathbf{\Phi}_{0s}^T$ and utilizing Eqs. (13) yields the relation

$$\mathbf{\Phi}_{0s}^T B_1\mathbf{\Phi}_{0r} + \mathbf{\Phi}_{0s}^T B_0\mathbf{\Phi}_{1r} = -\lambda_{0r}\mathbf{\Phi}_{0s}^T A_1\mathbf{\Phi}_{0r} - \lambda_{0r}\mathbf{\Phi}_{0s}^T A_0\mathbf{\Phi}_{1r} - \lambda_{1r}\delta_{rs} \tag{16}$$

It can be observed from Eqs. (14) that if A_1 and B_1 are zero, then $\lambda_{1r} = 0$ ($r = 1,2,...,2n$) and $\mathbf{\Phi}_{1r}$ become scalar multiples of $\mathbf{\Phi}_{0r}$. In general, $\mathbf{\Phi}_{1r}$ can be written as a linear combination of the vectors $\mathbf{\Phi}_{01},\mathbf{\Phi}_{02},...,\mathbf{\Phi}_{02n}$. To guarantee that $\mathbf{\Phi}_{1r} = 0$, when A_1 and B_1 are zero, it is assumed that the perturbation eigenvector has the form[19]

$$\mathbf{\Phi}_{1s} = \sum_{k=1}^{2n} \varepsilon_{sk}\mathbf{\Phi}_{0k}, \quad \varepsilon_{ss} = 0, \quad s = 1,2,...,2n \tag{17}$$

Using Eq. (17) in Eq. (16) and letting $s = r$ result in an expression for the perturbed eigenvalues

$$\lambda_{1r} = -\mathbf{\Phi}_{0r}^T[\lambda_{0r}A_1 + B_1]\mathbf{\Phi}_{0r}, \qquad r = 1,2,...,2n \tag{18}$$

In the sequel, it will be shown that Eq. (18) can be used as the central equation for identification of the damping matrix.

For a lightly damped structure, the frequency response function closely resembles that of the corresponding undamped structure, except near the resonant peaks. Therefore, given a set of complex frequency response measurements from a lightly damped structure, identification of the mass and stiffness matrices can be performed, as described in the previous section, by using the real components of the frequency response measurements and the imaginary components of the eigenvalue measurements. Again, this method will only be accurate for frequency response measurements away from the resonant peaks. Once the mass and stiffness matrices have been identified, the damping matrix can be determined as follows. First, the perturbed eigenvalues λ_{1r} are obtained by simply subtracting the undamped modeled eigenvalues λ_{0r} from the measured eigenvalues $\tilde{\lambda}_r$

$$\lambda_{1r} = \tilde{\lambda}_r - \lambda_{0r} = (\tilde{\sigma}_r + i\tilde{\omega}_r) - (i\omega_{0r}) \tag{19a}$$

$$\lambda_{1r} = \tilde{\sigma}_r + i(\tilde{\omega}_r - \omega_{0r}) \tag{19b}$$

where $\tilde{\sigma}_r$ and $\tilde{\omega}_r$ are the rth measured damping term and damped natural frequency. To utilize Eq. (18), the eigenvectors $\mathbf{\Phi}_{0r}$ must be normalized according to Eqs. (13). In general, the form of the eigenvectors becomes

$$\mathbf{\Phi}_{0r} = \begin{Bmatrix} \phi_{0r} \\ i\omega_{0r}\phi_{0r} \end{Bmatrix} \tag{20}$$

where ϕ_{0r} are the mode shapes from the identified undamped model, normalized with respect to the identified mass matrix. Therefore, using Eq. (20) in Eq. (13a) determines scale factors α_r necessary to normalize $\mathbf{\Phi}_{0r}$, such that Eq. (13) is satisfied

$$\alpha_r\{\phi_{0r}^T \; i\omega_{0r}\phi_{0r}^T\}\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} \phi_{0r} \\ i\omega_{0r}\phi_{0r} \end{Bmatrix} = 1 \tag{21a}$$

or

$$\alpha_r(-\phi_{0r}^T K\phi_{0r} - \omega_{0r}^2\phi_{0r}^T M\phi_{0r}) = 1 \tag{21b}$$

or

$$\alpha_r = -\frac{1}{2\omega_{0r}^2} \tag{21c}$$

The normalized eigenvectors $\mathbf{\Phi}_{0r}$ can now be written as

$$\mathbf{\Phi}_{0r} = \frac{i}{\sqrt{2}\omega_{0r}}\begin{Bmatrix} \phi_{0r} \\ i\omega_{0r}\phi_{0r} \end{Bmatrix} \tag{22}$$

Expanding the damping matrix in a similar fashion to Eqs. (8), and using Eqs. (22) and (19b) in Eq. (18) with $A_1 = 0$ and B_1 defined in Eq. (11), yields the relation

$$\tilde{\sigma}_r + i(\tilde{\omega}_r - \omega_{0r}) = -\tfrac{1}{2}\phi_{0r}^T\left[\hat{C} + \sum_{q=1}^{R}\xi_q C_q\right]\phi_{0r} \tag{23}$$

where $\hat{C}$ is the initial damping matrix, C_q the qth damping submatrix, ξ_q the qth damping submatrix scale factor, and R the total number of damping submatrices. Because the right side of Eq. (23) is real, the first-order perturbation solution does not predict a change in the undamped natural frequencies due to the addition of the light damping and, therefore, only the measured damping terms $\tilde{\sigma}_r$ are used to identify the damping matrix. Rearranging Eq. (23) to solve for the submatrix scale factors yields the linear least-squares problem

$$\begin{Bmatrix} -2\tilde{\sigma}_1 - \phi_{01}^T\hat{C}\phi_{01} \\ \cdot \\ \cdot \\ -2\tilde{\sigma}_m - \phi_{0m}^T\hat{C}\phi_{0m} \end{Bmatrix} = \begin{bmatrix} \phi_{01}^T C_1\phi_{01} & \cdots & \phi_{01}^T C_R\phi_{01} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \phi_{0m}^T C_1\phi_{0m} & \cdots & \phi_{0m}^T C_R\phi_{0m} \end{bmatrix}\begin{Bmatrix} \xi_1 \\ \cdot \\ \cdot \\ \xi_R \end{Bmatrix} \tag{24}$$

where it is assumed that there are m ($>R$) measured damping terms. Solving Eq. (24) for the submatrix scale factors leads to the desired damping matrix. The advantages of this perturbation approach are twofold: Identification of the damped equations of motion can be performed in configuration space without the need to solve the state-space eigenvalue problem, and the original damping matrix $\hat{C}$ need only represent the true damping matrix in the coupling of the elements. (Due to the linearity of the equations, the original numerical values can be off by orders of magnitude.) In most practical applications, the form of the initial damping matrix $\hat{C}$ will not be known. However, if an assumed form is prescribed (from a Rayleigh dissipation function, for example) Eq. (24) can still be used to provide a best fit (in a least-squares sense) to the measured damping terms $\tilde{\sigma}_r$.

Example 1

The mass and stiffness matrices are identified for the simple manipulator arm, shown in Fig. 1. The structure consists of two flexible appendages, rotational springs at the base and at the connecting joint, and a grip with mass and inertia.

The estimation process is initiated using approximate mass and stiffness matrices obtained by increasing the true mass properties by 10% and decreasing the true stiffness properties by 10%. The first five natural frequencies of the true model and the frequency response function representing the ratio of transverse displacement at the connecting joint to torque at the base

Table 1 Group modal energy distributions for the simple manipulator arm

	Kinetic energy (%) of mass groups				Potential energy (%) of stiffness groups			
Mode	I	II	III	IV	I	II	III	IV
1	0.0	0.5	0.0	99.5	0.2	0.1	66.5	33.2
2	58.8	39.1	1.9	0.2	0.1	0.1	33.3	66.5
3	27.5	47.3	25.2	0.0	19.5	80.5	0.0	0.0
4	64.0	22.1	13.9	0.0	75.7	24.1	0.1	0.1
5	16.5	38.8	44.7	0.0	7.4	92.6	0.0	0.0

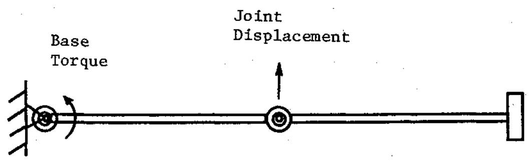

Fig. 1 Simple manipulator arm.

Table 2 Free-response identification results for the simple manipulator arm

Mode	$\tilde{\omega}_n$	ω_{n_o}	ω_{nf}
1	0.0230 rad/s	0.0209 rad/s	0.0230 rad/s
2	1.062	0.9617	1.066
3	55.44	50.15	55.55
4	91.12	82.42	91.19
5	156.75	141.78	156.98

are treated as measurements. The mode shapes from the approximate initial model are used for "measured" mode shapes. To cast the model in terms of mass and stiffness submatrices, the following mass and stiffness element groups are chosen:

Mass:
- Group I Mass matrix contribution of appendage 1
- Group II Mass matrix contribution of appendage 2
- Group III Tip mass moment of inertia
- Group IV Tip mass

Stiffness:
- Group I Stiffness matrix contribution of appendage 1
- Group II Stiffness matrix contribution of appendage 2
- Group III Base rotational stiffness
- Group IV Joint rotational stiffness

The initial fractional modal energy contributions from each element group (obtained from $\phi_0^T M_r \phi_0$ and $\phi_0^T K_r \phi_0 / \omega_r^2$) are given in Table 1. It is apparent from examination of the potential energy distribution that the first two modes approximate those that would be obtained for a two degree-of-freedom model with rigid appendages and that the higher modes represent the flexibility of the appendages. As a consequence of this observation, a two-step process was used to identify the structure. First, the three highest modes were used to identify mass element groups I, II, and III and stiffness element groups I and II. Then, the two lowest modes were used to identify mass element group IV and stiffness element groups III and IV. The free- and forced-response identification results (after two iterations) are provided in Table 2 and Fig. 2, respectively.

Example 2

The mass, damping, and stiffness matrices are identified for the planar truss structure shown in Fig. 3. Both internal (mate-

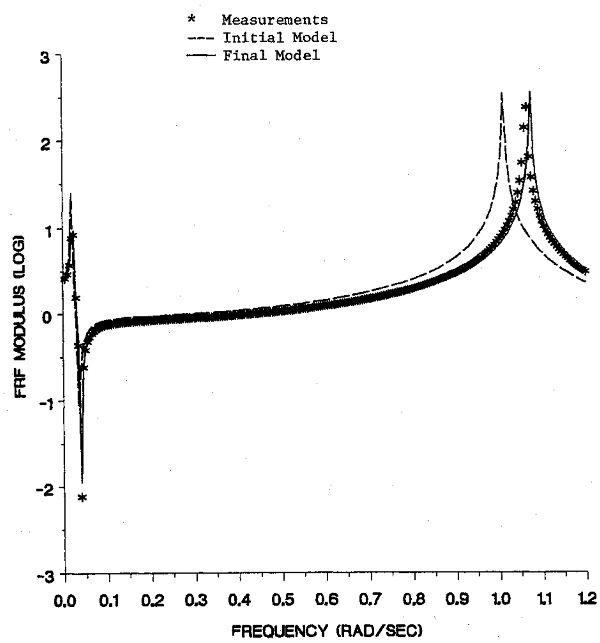

Fig. 2a Low-range frequency response results for the simple manipulator arm.

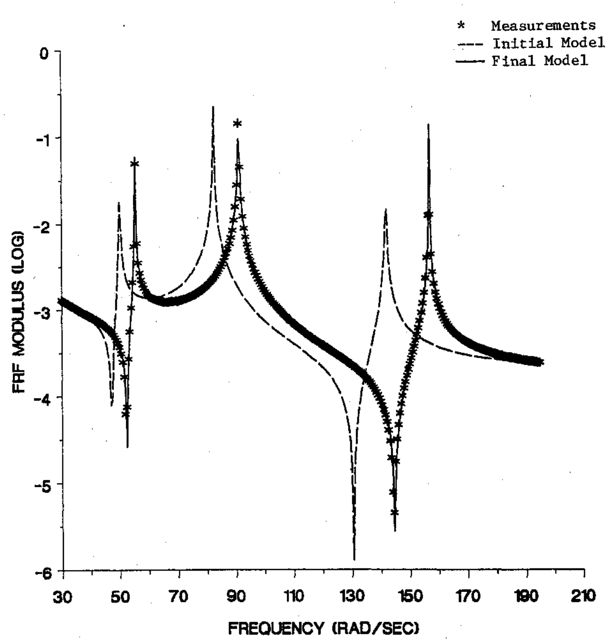

Fig. 2b High-range frequency response results for the simple manipulator arm.

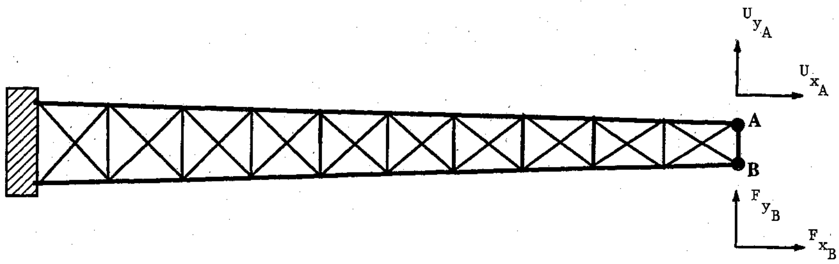

Fig. 3 Planar truss structure.

Table 3 Free-response identification results for the planar truss structure

Mode	$\tilde{\lambda}$	λ_0	λ_f
1	$-0.0872+6.73i$	$6.11i$	$-0.0872+6.93i$
2	$-0.0890+34.59i$	$31.45i$	$-0.0888+35.46i$
3	$-0.0891+87.58i$	$78.89i$	$-0.0898+88.29i$
4	$-0.1042+117.11i$	$106.67i$	$-0.1039+119.39i$
5	$-0.0907+156.49i$	$142.12i$	$-0.0906+157.86i$
6	$-0.0914+240.33i$	$217.54i$	$-0.0915+240.05i$
7	$-0.0929+332.09i$	$302.00i$	$-0.0925+331.39i$
8	$-0.1039+359.29i$	$325.25i$	$-0.1042+363.28i$

rial) and external (atmospheric) light viscous damping is present, although the magnitude of damping is unknown.

The measurement set consists of the first eight complex eigenvalues and transverse and longitudinal frequency response functions between points A and B. To simulate measurement errors, the eigenvalue and frequency response measurements are corrupted with random Gaussian noise (zero mean, $SD_\lambda = 0.01|\lambda_r|$, $SD_h = 0.11|h_{jk}|$). As in Example 1, the mode shapes from the initial model are chosen for "measured" mode shapes. To cast the model in terms of submatrices, the following element groups are chosen:

Mass and stiffness:
- Group I 20 upper and lower bending elements
- Group II 20 diagonal bending/shear elements

Damping:
- Group I External viscous damping matrix
- Group II Internal viscous damping matrix

Preliminary examination of the modal kinetic and potential energy distributions indicates that the vertical truss members contribute no energy to the first eight modes and are, therefore, not used for model improvement.

The mass and stiffness matrices are approximated initially by increasing the true mass properties by 10% and decreasing the true stiffness properties by 10%. The initial approximations of the external and internal passive damping matrices are only accurate in the coupling of the elements (the numerical values are off by orders of magnitude). The identification process requires two steps: 1) identification of the mass and stiffness matrices from the real components of the measured frequency response functions and the imaginary components of the measured eigenvalues, and 2) identification of the damping matrix from the real components of the measured eigenvalues. The free- and forced-response identification results for the structure are presented in Table 3 and Fig. 4, respectively.

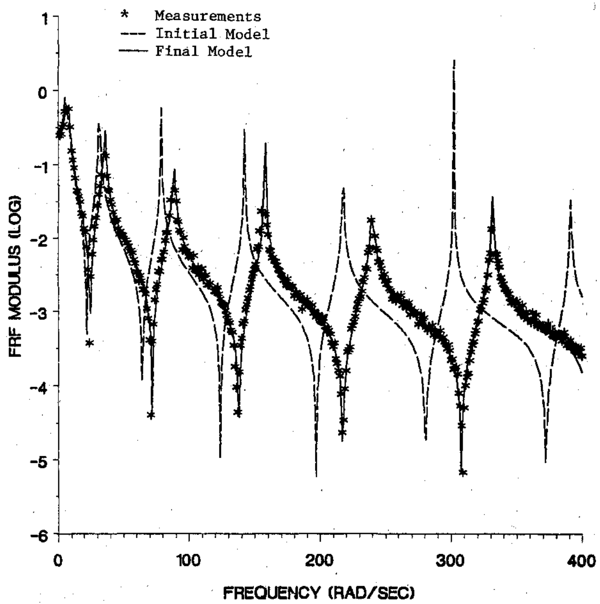

Fig. 4a Transverse frequency response results for the planar truss structure.

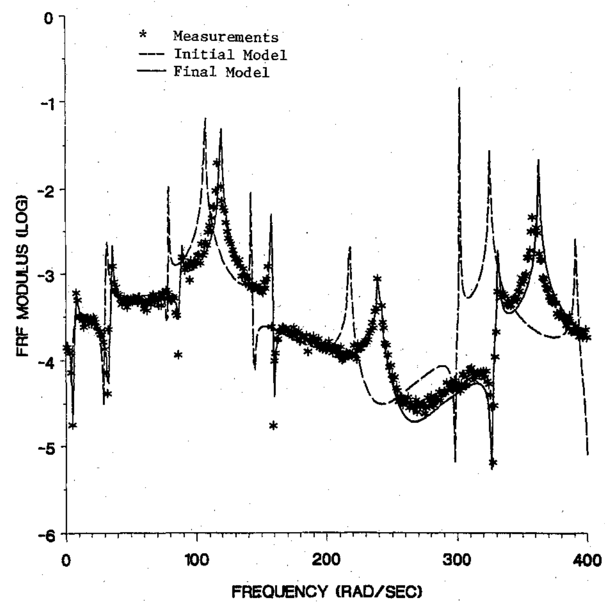

Fig. 4b Longitudinal frequency response results for the planar truss structure.

Conclusions

A method for updating initial mathematical models of undamped and lightly damped linear structures has been presented and successfully tested in two simulated examples. The advantages of the method are the following:

1) Incorporation of measured frequency response functions provides a unique scaling of prescribed submatrices of an initial mathematical model of the structure.

2) All least-squares formulations are linear.

3) The consistency of the original model is maintained. (No unmodeled coupling occurs as a consequence of the identification process.)

4) Use of submatrix scale factors can potentially limit the identification process to a relatively small set of parameters.

5) For lightly damped structures, the identification can be performed in the n configuration space without the need to solve the $2n$ state-space eigenproblem.

6) For lightly damped structures, the original estimate of the damping matrix need only be accurate in the coupling of the elements. (The numerical values can be off by orders of magnitude.) However, if the original form of the damping matrix is unknown, the method will still provide a unique scaling of an assumed initial (symmetric) damping matrix and its submatrices to best fit measured damping terms.

It should be noted that the examples presented in this paper considered only those structures with widely spaced, well-defined modes. Further research is needed to address the applicability of the method to actual structures containing dense modal spectra and in the presence of real measurement and modeling errors.

References

[1]Hendricks, S. L., Hayes, S. M., and Junkins, J. L., "Structural Parameter Identification for Flexible Spacecraft," AIAA Paper 84-0060, Jan. 1984.

[2]Creamer, N. G. and Hendricks, S. L., "Structural Parameter Identification Using Modal Response Data," *Proceedings of the Fifth VPI&SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft*, edited by L. Meirovitch, AIAA, New York, 1985.

[3]Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, Feb. 1974, pp. 185-190.

[4]Dobbs, M. W. and Nelson, R. B., "Parameter Identification of Large Structural Models—Concept and Reality," ASME Winter Annual Meeting, Boston, MA, Nov. 1983.

[5]Hasselman, T. K., "A Perspective on Dynamics Model Verification," ASME Winter Annual Meeting, Boston, MA, Nov. 1983.

[6]Martinez, D. R., "Estimation Theory Applied to Improving Dynamic Structural Models," Sandia National Lab., Albuquerque, NM, Sandia Rept. SAND82-0572, 1984.

[7]Berman, A. and Flannely, W. G., "Theory of Incomplete Models of Dynamic Structures," *AIAA Journal*, Vol. 9, Aug. 1971, pp. 1481–1487.

[8]Berman, A. and Nagy, E. J., "Improvement of Large Analytical Model Using Test Data," *AIAA Journal*, Vol. 21, Aug. 1983, pp. 1168–1173.

[9]Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," *AIAA Journal*, Vol. 17, Oct. 1979, pp. 1147–1148.

[10]Wei, F. S., "Stiffness Matrix Corrections from Incomplete Test Data," *AIAA Journal*, Vol. 18, Oct. 1980, p. 1274.

[11]Chen, J. C., Kuo, C. P., and Garba, J. A., "Direct Structural Parameter Identification by Modal Test Results," *Proceedings of the AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics, and Materials Conference*, Part 2, AIAA, New York, 1983.

[12]White, C. W. and Maytum, B. D., "Eigensolution Sensitivity to Parametric Model Perturbations," *Shock and Vibration Bulletin*, Bulletin 46, Part 5, Aug. 1976, pp. 123–133.

[13]Rajaram, S., "Identification of Vibration Parameters of Flexible Structures," Ph.D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA, May 1984.

[14]Rajaram, S. and Junkins, J. L., "Identification of Vibrating Flexible Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 8, July–Aug. 1985, pp. 463–470.

[15]Hendricks, S. L., Rajaram, S., Kamat, M. P., and Junkins, J. L., "Identification of Large Flexible Structures Mass/Stiffness and Damping From On-Orbit Experiments," *Journal of Guidance, Control, and Dynamics*, Vol. 7, March–April 1984, pp. 244–245.

[16]Ewins, D. J., *Modal Testing: Theory and Practice*, Research Studies Press, England, 1984.

[17]Ewins, D. J. and Gleeson, P. T., "A Method for Modal Identification of Lightly Damped Structures," *Journal of Sound and Vibration*, Vol. 84, 1982, pp. 57–79.

[18]Juang, J. N., "Mathematical Correlation of Modal Parameter Identification Methods Via System Realization Theory," NASA 87720, April 1986.

[19]Meirovitch, L., *Computational Methods in Structural Dynamics*, Sijthoff & Noordhoff, Rockville, MD, 1980.